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SUBLIMATION OF A SOLID NEAR A CRITICAL POINT IN FLAT AND AXISYMMETRICAL GAS FLOWS

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Аннотация—В работе получено точное решение задачи о сублимации твердого тела в окрестности критической точки в плоском и осесимметричном потоках несжимаемого газа. Установлены конечные соотношения для определения концентрации и температуры на волне сублимации.

Аналогичную задачу рассматривает Робертс [1]. Однако, им строится приближённое решение с использованием интегрального метода Польгаузена (распределение функций в пограничном слое задаётся).

NOMENCLATURE

x, y,	co-ordinates;
u, v,	velocity components;
t,	time;
ρ, P,	density and pressure;
$\mu, \lambda, \mathscr{D}_{1,2}$,	coefficients of viscosity, thermal conductivity and diffusion;
c_p,	specific heat capacity;
$\tilde{\mathscr{D}}$,	velocity of sublimation front propagation;
l,	latent sublimation heat;
$\sigma = \frac{c_p \mu}{\lambda}$,	Prandtl number;
$Sc = \frac{\mu}{\rho \mathscr{D}_{1,2}}$,	Schmidt number;
$Le = \frac{\lambda}{\rho c_p \mathscr{D}_{1,2}}$,	Lewis number;
$H = h + \frac{u^2}{2}$,	complete enthalpy;
$h = Ch_a + (1 - C)/h_b$,	mixture heat content;
C,	mass concentration of sublimating vapours in gas mixture;
β,	proportionality coefficient in law of external flow velocity;
R,	gas constant.

Subscripts

1,	in a solid;
a, b,	pertaining respectively to vapour and gas;
0,	on sublimation wave;
B,	at moment of boiling.

1. STATEMENT OF PROBLEM. SYSTEM OF ORDINARY DIFFERENTIAL EQUATIONS

THE interaction of a gas flow and solid near a critical point, resulting in the sublimation of the solid surface, is considered. The sublimation front is assumed to propagate into the body with constant velocity $\tilde{\mathscr{D}}$: i.e. a stationary sublimation regime is considered.

Equations for a gas and solid are considered to be non-stationary. The solution is given for a solid in a flat and axisymmetrical gas flow.

A heat transfer process takes place both in the gas and the solid. The equation of a gas and solid should therefore be solved simultaneously combining corresponding solutions on a sublimation wave.

The consideration of a diffusion equation is caused by the fact that a bi-component mixture (vapour–gas) is formed during the sublimation of a solid surface.

The problem is solved by assuming a laminar boundary layer. To simplify this problem the properties of the gas and solid are assumed to be constant.

The following system of co-ordinates is chosen: axis x is directed along the surface of the solid, the critical point being taken as the origin; axis y is normal to the surface of the solid, and is directed towards the gas-side, the value being read from the initial boundary between the gas and solid (Fig. 1).

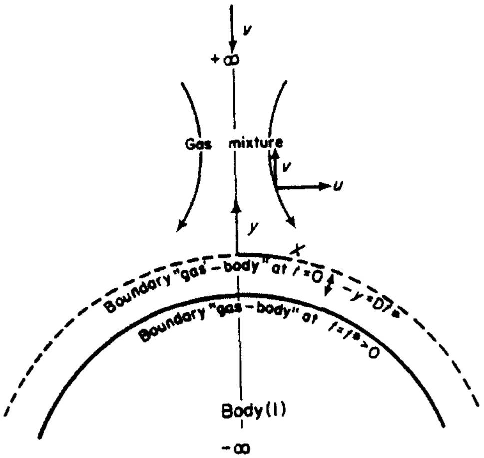

FIG. 1.

The problem considered is described by an original system of equations consisting of equations of continuity, motion, diffusion and energy for a gas mixture.

$$\left.\begin{aligned}&\frac{1}{x^{n-1}}\frac{\partial(ux^{n-1})}{\partial x}+\frac{\partial v}{\partial y}=0;\\&\rho\frac{\mathrm{d}u}{\mathrm{d}t}=-\frac{\partial P}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2}\\&\frac{\partial P}{\partial y}=0;\\&\rho\frac{\mathrm{d}C}{\mathrm{d}t}=\rho\mathscr{D}_{1,2}\frac{\partial^2 C}{\partial y^2};\\&\rho\frac{\mathrm{d}H}{\mathrm{d}t}=\frac{\mu}{\sigma}\frac{\partial}{\partial y}\left[\frac{\partial H}{\partial y}\right.\\&\qquad+(\sigma-1)\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\\&\qquad\left.+\left(\frac{1}{Le}-1\right)(h_a-h_b)\frac{\partial C}{\partial y}\right].\end{aligned}\right\}\quad(1.1)$$

and the equation of thermal conductivity in a solid:

$$\rho_1 c_1\frac{\partial T_1}{\partial t}=\lambda_1\frac{\partial^2 T}{\partial y^2}.\qquad(1.2)$$

In the continuity equation, $n = 1$ for a plane case, $n = 2$ for an axisymmetrical case.

Taking into account that in the vicinity of the critical point the Mach number is small, and assuming that the Lewis number is equal to unity $Le = 1$, the energy equation becomes simplified:

$$\rho\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mu}{\sigma}\frac{\partial^2 h}{\partial y^2}.\qquad(1.3)$$

The following system of boundary conditions corresponds to equations (1.1), (1.2) and (1.3):

(a) on the external side of the boundary layer:

$$y\to\infty,\quad u=u_\infty=\beta x,\quad h=h_\infty,\quad C=0;$$

(b) on the sublimation wave:

$$\left.\begin{aligned}&y=\tilde{\mathscr{D}}t,\quad u=0,\quad \rho(\tilde{\mathscr{D}}-v)=\rho_1\tilde{\mathscr{D}},\\&(\tilde{\mathscr{D}}-v)(1-C)=\mathscr{D}_{1,2}\frac{\partial C}{\partial y},\\&T=T_1,\quad \rho(\tilde{\mathscr{D}}-v)\,l=\lambda_1\frac{\partial T_1}{\partial y}-\lambda\frac{\partial T}{\partial y},\\&C=\left[1+\left(e^{\epsilon/R[(1/T_0)-(1/T_B)]}-1\right)\frac{m_b}{m_a}\right]^{-1}.\end{aligned}\right\}(1.4)$$

(the latter relation is the curve of vapour pressure on the sublimation wave);

(c) at infinity in the solid:

$$y\to-\infty,\quad T=T_{-\infty}.$$

Since the stationary sublimation regime is determined below, the initial conditions are not presented.

The second last condition on the sublimation wave may be transformed. The use of both the obvious relation:

$$\lambda\frac{\partial T}{\partial y}=\frac{\lambda}{c_p}\frac{\partial h}{\partial y}-\frac{\lambda}{c_p}(h_a-h_b)\frac{\partial C}{\partial y}$$

and the condition of concentration conservation on the sublimation wave gives:

$$\rho(\tilde{\mathscr{D}}-v)\left[l-(h_a-h_b)(1-C)\right]=\lambda_1\frac{\partial T_1}{\partial y}-\frac{\lambda}{c_p}\frac{\partial h}{\partial y}.\qquad(1.5)$$

Let us transform a system of equations in partial derivatives (1.1), (1.2) to the system of ordinary differential equations, and introduce an independent variable:

$$\eta = \sqrt{\left(\frac{u_\infty}{\nu_x}\right)}\,(y - \tilde{\mathscr{D}}t) = \sqrt{\left(\frac{\beta}{\nu}\right)}\,(y - \tilde{\mathscr{D}}t). \quad (1.6)$$

The solution of the problem is sought for in the form:

$$\left.\begin{aligned} &u = \beta x f'(\eta);\ v = -n\sqrt{(\nu\beta)} f(\eta) + \tilde{\mathscr{D}};\\ &h = h_{-\infty} + (h_\infty - h_{-\infty})\,\theta(\eta);\\ &T_1 = T_{-\infty} + (T_\infty - T_{-\infty})\,\theta_1(\eta). \end{aligned}\right\}(1.7)$$

With the aid of the transformations, equations (1.6) and (1.7), the initial system of equations is reduced as follows:

$$\left.\begin{aligned} &f'^2 - nff'' = 1 + f''',\\ &-nf\frac{dC}{d\eta} = \frac{1}{\sigma}\,\frac{d^2C}{d\eta^2},\\ &-nf\frac{d\theta}{d\eta} = \frac{1}{\sigma}\,\frac{d^2\theta}{d\eta^2},\\ &-\mathscr{D}\frac{d\theta_1}{d\eta} = \frac{1}{\sigma_1}\,\frac{d^2\theta}{d\eta^2}, \end{aligned}\right\}(1.8)$$

where

$$\mathscr{D} = \frac{\tilde{\mathscr{D}}}{\sqrt{\beta\nu}}, \quad \sigma_1 = \sigma\,\frac{c_1}{c_p}\cdot\frac{\rho_1}{\rho}\cdot\frac{\lambda}{\lambda_1}.$$

The boundary conditions in the variables of equations (1.6) and (1.7) can be written:

$$\eta \to \infty,\quad f'(\infty) = 1,\quad \theta = 1,\quad C = 0,$$

$$\left.\begin{aligned} &\eta = 0,\quad f'(0) = 0,\quad nf(0) = \frac{\rho_1}{\rho}\mathscr{D},\\ &nf(0)\,(1 - C)\,\sigma = \frac{dC}{d\eta},\\ &T = T_1,\ n\sigma f(0)\,\frac{l - (h_a - h_b)\,(1 - C)}{h_\infty - h_{-\infty}}\\ &= \frac{\lambda_1}{\lambda}\,\frac{d\theta_1}{d\eta} - \frac{d\theta}{d\eta},\\ &C = \left[1 + (e^{e/R[(1/T_0) - (1/T_B)]} - 1)\,\frac{m_b}{m_a}\right]^{-1},\\ &\eta \to -\infty,\quad \theta_1 = 0. \end{aligned}\right\}(1.9)$$

2. DETERMINATION OF CONCENTRATION AND TEMPERATURE ON THE SUBLIMATION WAVE

From the heat conductivity equation for a solid it is easy to obtain the relation:

$$\frac{d\theta}{d\eta} = -\,\sigma_1\mathscr{D}\theta_1(0). \quad (2.1)$$

Equation (2.1) is used for transforming the condition of energy balance on the sublimation wave. After simple transformations we obtain:

$$-\frac{d\theta}{d\eta} = \sigma f(0) n\, \frac{l - (h_a - h_b)\,(1 - C_0) + c_1(T_0 - T_{-\infty})}{h_\infty - h_{-\infty}}. \quad (2.2)$$

The boundary problem, equations (1.8) and (1.9), is then reduced to the following:

$$\left.\begin{aligned} &f' - nff'' = 1 + f''';\\ &-nf\frac{dC}{d\eta} = \frac{1}{\sigma}\,\frac{d^2C}{d\eta^2};\\ &-nf\frac{d\theta}{d\eta} = \frac{1}{\sigma}\,\frac{d^2\theta}{d\eta^2} \end{aligned}\right\}(2.3)$$

under the boundary conditions

$$\left.\begin{aligned} &\eta \to \infty,\quad f'(\infty) = 1,\quad \theta = 1,\quad C = 0,\\ &\eta = 0,\quad f'(0) = 0,\quad nf(0) = \frac{\rho_1}{\rho}\mathscr{D},\\ &nf(0)\,(1 - C)\,\sigma = \frac{dC}{d\eta},\\ &-\frac{d\theta}{d\eta} = n\sigma f(0)\ \times\\ &\frac{l - (h_a - h_b)\,(1 - C_0) + c_1(T_0 - T_{-\infty})}{h_\infty - h_{-\infty}},\\ &C = \left[1 + (e^{e/R[(1/T_0) - (1/T_B)]} - 1)\,\frac{m_b}{m_a}\right]^{-1}. \end{aligned}\right\}(2.4)$$

It is easy to see that the second and third equations may be easily integrated, function $f(\eta)$ still remaining unknown.

Upon integration of the equations mentioned and determination of the constants of the boundary conditions, the expression for concentration and heat content distribution in a gas mixture is obtained:

$$\left.\begin{aligned}C(\eta) &= n\sigma f(0)\frac{\phi(\eta)-\phi(\infty)}{1-\phi(\infty)\sigma f(0)n};\\ \theta(\eta) &= n\sigma f(0)\\ &\times\frac{l-(h_a-h_b)(1-C_0)+c_1(T_0-T_{-\infty})}{h_\infty-h_{-\infty}}\\ &\qquad\times[\phi(\infty)-\phi(\eta)]+1,\end{aligned}\right\}(2.5)$$

where

$$\phi(\eta)=\int_0^\eta e^{-\sigma n\int_0^t f(t)dt}\,.\,dt.$$

From equation (2.5) the expression for concentration and heat content on the sublimation wave may be easily obtained:

$$\left.\begin{aligned}C_0 &= -\frac{n\sigma f(0)\phi(\infty)}{1-n\sigma f(0)\phi(\infty)};\\ \theta_0 &= n\sigma f(0)\phi(\infty)\\ &\times\frac{l-(h_a-h_b)(1-C_0)+c_1(T_0-T_{-\infty})}{h_\infty-h_{-\infty}}.\end{aligned}\right\}(2.6)$$

Eliminating from equation (2.5) the combination of values $n\sigma f(0)\phi(\infty)$, where $f(0)$ and $\phi(\infty)$ are still unknown, the finite relation connecting concentration and heat values on the sublimation wave is obtained:

$$\theta(0)=\frac{h_0-h_{(-\infty)}}{h_\infty-h_{(-\infty)}}=1+\frac{C_0}{C_0-1}\times\frac{l-(h_a-h_b)(1-C_0)+c_1(T_0-T_{-\infty})}{h_\infty-h_{-\infty}}. \quad (2.7)$$

The heat content of a mixture may be expressed through the heat contents of components:

$$h_0=C_0h_a+(1-C_0)h_b, \quad (2.8)$$

then equation (2.7) is reduced to the form:

$$\frac{l+c_1(T_0-T_{-\infty})}{h_b(0)-h(\infty)}=\frac{C_0-1}{C_0}. \quad (2.9)$$

Adding to equation (2.9) the equation of the vapour pressure curve on the sublimation wave, the system of two finite equations with two unknowns, which is easily solved, may be obtained.

Thus, to obtain temperature T_0 and concentration C_0 on the sublimation wave it is necessary to solve the system of equations:

$$\frac{l+c_1(T_0-T_{-\infty})}{h_b(0)-h_\infty}=\frac{C_0-1}{C_0};$$

$$C_0=\left[1+\left(e^{\epsilon/R[(1/T_0)-(1/T_B)]}-1\right)\frac{m_b}{m_a}\right]^{-1}. \quad (2.10)$$

3. DETERMINATION OF VELOCITY OF SUBLIMATION FRONT PROPAGATION

In addition to concentration and heat content values on the sublimation wave which are, in fact, determined, it is necessary to know the velocity of propagation of the sublimation front.

From the first equation (2.6) it is easy to obtain:

$$\frac{C_0}{C_0-1}=n\sigma f(0)\phi(\infty)$$

$$=n\sigma f(0)\int_0^\infty e^{-n\sigma\int_0^t f(t)dt}\,.\,dt. \quad (3.1)$$

If the dependence of $\phi(\infty)$ upon $f(0)$ were known, then equation (3.1) would give the relation between concentration on the sublimation wave and the dimensionless velocity of propagation of the sublimation front $f(0)$. Then, by the known C_0 it would be easy to find $f(0)$, and the problem would be completely solved.

Thus, the equation of motion should be considered, to find the dependence of $\phi(\infty)$ upon $f(0)$:

$$f'^2-nff''=f'''+1 \quad (3.2)$$

with the boundary conditions

$$f(0)=\alpha,\quad f'(0)=0,\quad f'(\infty)=1.$$

Solving numerically the boundary problem, equation (3.2), we obtain the dependence of $f''(0)$ upon $f(0)=\alpha$, the form of which for plane and axisymmetrical cases is presented in Fig. 2.

Knowing the dependence of $f''(0)$ upon $f(0)$, we get the dependence of the function $f(\eta)$ upon $f(0)$ in the form of the Maclaurin series:

$$f(\eta) = \alpha + \frac{a}{2}\eta^2 - \frac{n\alpha a + 1}{3!}\eta^3 + \frac{n\alpha(n\alpha a + 1)}{4!}\eta^4 + \frac{(2-n)\,a^2 - n^2\alpha^2\,(n\alpha a + 1)}{5!}\eta^5$$
$$+ \frac{(-8+5n)\,n\alpha a^2 + (-6+4n+n^4\alpha^4)\,a + n^3\alpha^3}{6!}\eta^6$$
$$+ \frac{(22-16n)\,n^2\alpha^2 a^2 + (26-19n-n^4\alpha^4)\,n\alpha a + (6-4n-n^4\alpha^4)}{7!}\eta^7 + \ldots, \qquad (3.3)$$

where $\alpha = f(0); \quad a = f''(0).$

To find the dependence of $J = n\sigma f(0)\phi(\infty)$ upon $\alpha = f(0)$, we evaluate the integral $\phi(\infty)$ for a set of values α:

$$\phi_\infty = \int_0^\infty e^{-n\sigma\int_0^t f(t)\mathrm{d}t}\,.\,\mathrm{d}t, \qquad (3.4)$$

applying the asymptotic method [2].

On integrating $f(\eta)$ and substituting the result into equation (3.4) we obtain:

$$\phi(\infty) = \int_0^\infty e^{-n\sigma\alpha\eta - n\sigma z(\eta)}\,.\,\mathrm{d}\eta, \qquad (3.5)$$

where

$$z(\eta) = \frac{a}{3!}\eta^3 - \frac{n\alpha a + 1}{4!}\eta^4 + \frac{n\alpha\,(n\alpha a + 1)}{5!}\eta^5 + \frac{(2-n)\,a^2 - n^2\alpha^2\,(n\alpha a + 1)}{6!}\eta^6$$
$$+ \frac{(-8+5n)\,n\alpha a^2 + (-6+4n+n^4\alpha^4)\,a + n^3\alpha^3}{7!}\eta^7$$
$$+ \frac{(22-16n)\,n^2\alpha^2 a^2 + (26-19n-n^4\alpha^4)\,n\alpha a + (6-4n-n^4\alpha^4)}{8!}\eta^8 + \ldots \qquad (3.6)$$

Changing the variable of integration from η to z, the limits of integration not being changed:

$$\phi(\infty) = \int_0^\infty e^{-n\sigma z}\, e^{-n\sigma\alpha\eta}\,.\,\frac{\mathrm{d}\eta}{\mathrm{d}z}\,.\,\mathrm{d}z. \qquad (3.7)$$

The integrand contains an unknown function of z:

$$e^{-n\sigma z\eta}\,.\,\frac{\mathrm{d}\eta}{\mathrm{d}z},$$

which will be sought for in the form of a series:

$$e^{-n\sigma\alpha\eta}\,.\,\frac{\mathrm{d}\eta}{\mathrm{d}z} = z^{-2/3}\sum_{m=0}^{\infty} d_m\, z^{m/3} \qquad (3.8)$$

The expansion coefficients d_m are determined using the known formula:

$$d_m = \frac{1}{6\pi_i}\oint \phi\,.\,\exp(-\alpha n\sigma\eta)\, z^{-(m+1)/3}\,.\,\mathrm{d}\eta \qquad (3.9)$$

From equation (3.9) it follows that d_m is $\frac{1}{3}$ of the coefficient of η^{-1} in the expansion of the function $\exp(-n\sigma\alpha\eta)\ z^{-(m+1)/3}$ in integral powers of η.

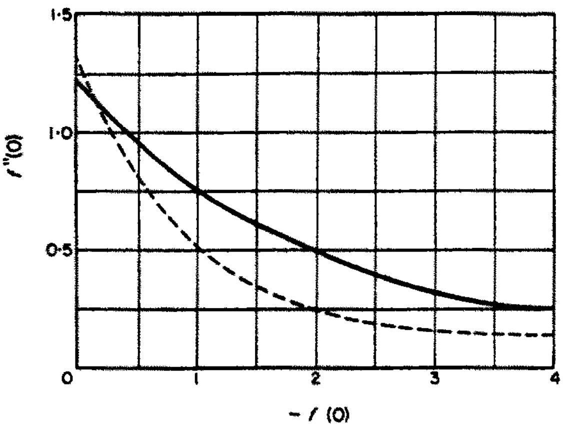

FIG. 2.
——— plane case;
— — — axisymmetrical case.

The expansion of $\exp(-n\sigma\alpha\eta)\, z^{-(m+1)/3}$ as a series is as follows:

$$\exp(-n\sigma\alpha\eta) = \sum_{m=0}^{\infty} (-1)^m \frac{(n\sigma\alpha\eta)^m}{m!}. \tag{3.10}$$

The expansion of $z^{-(m+1)/3}$ as a series in integral powers of η is obtained. For this, equation (3.6) is raised to the power $\frac{1}{3}$, and the expression obtained is expanded as a power series:

$$z^{-(m+1)/3} = \eta^{-(m+1)} (C_0 + C_1\eta + C_2\eta^2 + C_3\eta^3 + C_4\eta^4 + C_5\eta^5 + \ldots), \tag{3.11}$$

where

$$C_0 = \left(\frac{a}{3!}\right)^{-(m+1)/3};$$

$$C_1 = \frac{m+1}{3}\left(\frac{a}{3!}\right)^{-(m+4)/3} \frac{n\alpha a + 1}{4!};$$

$$C_2 = \frac{1}{2}\,\frac{m+1}{3}\left(\frac{a}{3!}\right)^{-(m+4)/3}\left\{\frac{m+4}{3}\left(\frac{a}{3!}\right)^{-1}\left(\frac{n\alpha a+1}{4!}\right)^2 - \frac{2}{5!}\,n\alpha\,(n\alpha a + 1)\right\};$$

$$C_3 = \frac{1}{3!}\,\frac{m+1}{3}\left(\frac{a}{3!}\right)^{-(m+4)/3}\left\{\frac{(m+4)(m+7)}{3^2}\left(\frac{a}{3!}\right)^{-2}\left(\frac{n\alpha a+1}{4!}\right)^3 - (m+4)\left(\frac{a}{3!}\right)^{-1}\frac{(n\alpha a+1)}{4!}\,\frac{2}{5!}\,n\alpha\,(n\alpha a+1) - \frac{3!}{6!}\left[(2-n)\alpha^2 - n^3\alpha^3 a - n^2\alpha^2\right]\right\};$$

$$C_4 = \frac{1}{4!}\,\frac{m+1}{3}\left(\frac{a}{3!}\right)^{-(m+4)/3}\left\{\frac{(m+4)(m+7)(m+10)}{3^3}\left(\frac{a}{3!}\right)^{-3}\left(\frac{n\alpha a+1}{4!}\right)^4 - 6\,\frac{(m+4)(m+7)}{3^2}\left(\frac{a}{3!}\right)^{-2}\left(\frac{n\alpha a+1}{4!}\right)^2\frac{2}{5!}\,n\alpha\,(n\alpha a+1) + (m+4)\left(\frac{a}{3!}\right)^{-1}\left[\frac{2}{5!}\,n\alpha\,(n\alpha a+1)\right]^2 - 4\,\frac{m+4}{3}\left(\frac{a}{3!}\right)^{-1}\frac{n\alpha a+1}{4!}\,\frac{3!}{6!}\left[(2-n)\alpha^2 - n^3\alpha^3 a - n^2\alpha^2\right] - \frac{4!}{7!}\left[(-8+5n)n\alpha\alpha^2 + (-6+4n+n^4\alpha^4)\alpha + n^3\alpha^3\right]\right\};$$

$$C_5 = \frac{1}{5!}\,\frac{m+1}{3}\left(\frac{a}{3!}\right)^{-(m+4)/3}\left\{\frac{(m+4)(m+7)(m+10)(m+13)}{3^4}\left(\frac{a}{3!}\right)^{-4}\left(\frac{n\alpha a+1}{4!}\right)^5 - 10\,\frac{(m+4)(m+7)(m+10)}{3^3}\left(\frac{a}{3!}\right)^{-3}\left(\frac{n\alpha a+1}{4!}\right)^3\cdot\frac{2}{5!}\,n\alpha(n\alpha a+1) + 15\,\frac{(m+4)(m+7)}{3^2}\left(\frac{a}{3!}\right)^{-2}\left(\frac{n\alpha a+1}{4!}\right)\left[\frac{2}{5!}\,n\alpha(n\alpha a+1)\right]^2 - 10\,\frac{(m+4)(m+7)}{3^2}\left(\frac{a}{3!}\right)^{-2} \times \left(\frac{n\alpha a+1}{4!}\right)^2\frac{3!}{6!}\left[(2-n)\alpha^2 - n^3\alpha^3 a - n^2\alpha^2\right] + 10\,\frac{(m+4)}{3}\left(\frac{a}{3!}\right)\frac{2}{5!}\,n\alpha(n\alpha a+1)\,\frac{3!}{6!} \times \left[(2-n)\alpha^2 - n^3\alpha^3 a - n^2\alpha^2\right] - 5\,\frac{m+4}{3}\left(\frac{a}{3!}\right)^{-1}\frac{n\alpha a+1}{4!}\,\frac{4!}{7!}\left[(-8+5n)\,n\alpha\alpha^2 + (-6+4n+n^4\alpha^4)\alpha + n^3\alpha^3 - \frac{5!}{8!}\left[(22-16n)n^2\alpha^2\alpha^2 + (26-19n-n^4\alpha^4)n\alpha\alpha + (6-4n-n^4\alpha^4)\right]\right\}.$$

Multiplying the right terms of equations (3.10) and (3.11), the expansion of

$$\exp(-n\sigma\alpha\eta) \,.\, z^{-(m+1)/3}$$

as a series may be obtained.

In the expansion obtained, $\frac{1}{3}$ of the coefficient of η^{-1} is equal to the coefficient a_m. Thus, coefficients d_m for a set of values $\alpha = f(0)$ are calculated.

Using z as the variable of integration, the integral $\phi(\infty)$ has the form:

$$\phi(\infty) = \int_0^\infty (d_0 + d_1 z^{1/3} + d_2 z^{2/3} + d_3 z + \dots)\, z^{-2/3} \,.\, e^{-n\sigma z} \,.\, dz. \quad (3.12)$$

On integrating we obtain:

$$\phi(\infty) = n\sigma f(0) \sum_{m=0}^{\infty} d_m (n\sigma)^{-(m+1)/3} \,.\, \Gamma\left(\frac{m+1}{3}\right) \quad (3.13)$$

where Γ is the gamma-function.

At the known coefficients d_m, the integral $\phi(\infty)$ may be easily calculated for a set of values $\alpha = f(0)$.

Hence, the dependence of the integral $\phi(\infty)$ upon $\alpha = f(0)$ is found. Then, from the equality, (3.1), the dependence of $C_0/(C_0 - 1)$ upon $\alpha = f(0)$ becomes known. This dependence is presented in Fig. 3. The curves are plotted for plane and axisymmetrical cases at Prandtl numbers 0·7 and 1.

Since the problem of determining the concentration C_0 on the sublimation wave is already solved (section 2), the dimensionless velocity of propagation of the sublimation front $f(0)$ is determined graphically from Fig. 2.

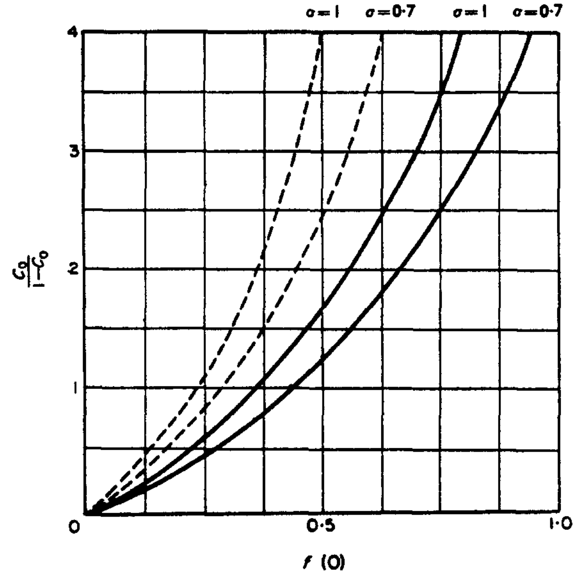

FIG. 3.
– – – – plane case;
———— axisymmetrical case.

The dimensional velocity of propagation of the sublimation front is expressed through $f(0)$ as follows:

$$\tilde{\mathscr{D}} = \frac{\rho}{\rho_1} \sqrt{(\beta\nu)}\, n f(0) \quad (3.14)$$

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Abstract—The paper gives an exact solution of a solid sublimation problem near a critical point in flat and axisymmetrical flows of an incompressible gas. Finite relations are established for determining concentration and temperature on a wave of sublimation. Plots for the velocity determination of sublimation front propagation are given.

An analogous problem is considered by Roberts [1]. However, he gives an approximate solution using the integral Pohlhausen method (the distribution of functions in a boundary layer is given).

Résumé—Les auteurs présentent une solution exacte du problème de la sublimation d'un solide au voisinage d'un point critique dans des écoulements plans et de révolution de gaz incompressible. Ils ont établi des relations finies permettant de déterminer la concentration et la température sur une onde de sublimation et donnent des courbes de la vitesse de propagation du front de sublimation.

Un problème analogue a été étudié par Roberts [1], mais ce dernier donne une solution approchée utilisant la méthode intégrale de Pohlhausen (la distribution des fonctions dans une couche limite est donnée).

Zusammenfassung—Für die Sublimation in der Nähe des kritischen Punktes im ebenen und achsensymmetrischen Strom eines nichtkompressiblen Gases wurde eine exakte Lösung ermittelt. Konzentration und Temperatur im Sublimationsbereich können nach angegebenen Beziehungen bestimmt werden. Aus Diagrammen ist die Geschwindigkeit, mit der die Sublimationsfront fortschreitet, zu entnehmen.

Ein analoges Problem ist von Roberts [1] bearbeitet worden. Mit Hilfe der integralen Pohlhausen-Methode erhält er eine Näherungslösung (bei gegebener Funktionsverteilung in der Grenzschicht).